Fracture Mechanics and Finite Element Analysis

Determining the basic parameters for a fracture mechanics analysis is easily accomplished using finite element modeling techniques. Nonetheless, the engineer must be cognizant of the limitations of both technologies for their rational use.

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Fracture mechanics and finite element analysis are 20th-century technologies that have a profound impact on the way engineers design mechanical devices, structures, and material systems. Although there is a wealth of literature in these two fields, the basic concepts of these technologies are simple.

Fracture mechanics describes the transfer of mechanical energy toward the creation of crack surfaces, i.e., the first law of thermodynamics. Finite element analysis is a numerical technique that solves continuum problems with an accuracy acceptable to engineers. Together these technologies provide powerful tools to predict critical loads or crack sizes that may cause fracture in proposed designs or existing structures. With the advent of modern personal computers and finite element codes it may now be much simpler to solve engineering fracture mechanics problems a priori rather than after a catastrophic fracture event. The U.S. Bureau of Mines is using such an approach to solve fracture problems in wear-resistant materials and to prevent premature rock failures in mining environments.

The foundation of fracture mechanics was laid in the 1920s by A.A. Griffith while he was working for the Royal Aircraft Establishment at Farnborough, U.K. According to J.E. Gordon [1], Griffith asked: "Why are there large variations between the strengths of different solids? Why don't all solids have the same strength? Why aren't they much stronger?"

The answer he found was that all materials contain some measurable size of cracks (e.g., very small voids). Griffith's idea was to relate the energy expended in the creation of new internal surfaces (i.e., cracks) to the change in strain energy of the structure. This was the crux of Griffith's work, that the fracture behavior of materials is controlled by an energy balance.

Based on some earlier work done by C.E. Inglis in 1913, Griffith showed that a crack may propagate if:

$$\frac{\partial}{\partial a}(S_e - U_e) = 0$$

where S_e is the energy required to create a new surface, U_e is the internal energy (such as strain and thermal) of the structure, and a is the crack length. This simple equation postulates that fracture behavior is balanced between a material parameter (the amount of energy expended to create a new surface) and a mechanics parameter (the amount of mechanical energy transferred into the structure). Using this equation it is possible to estimate (within an order of magnitude) the applied load or change in internal energy that will cause catastrophic failure as defined as:

$$\partial U_e/\partial a = \partial S_e/\partial a.$$

For example, if we have a thin rectangular plate (plane stress conditions) with a symmetrically located crack at

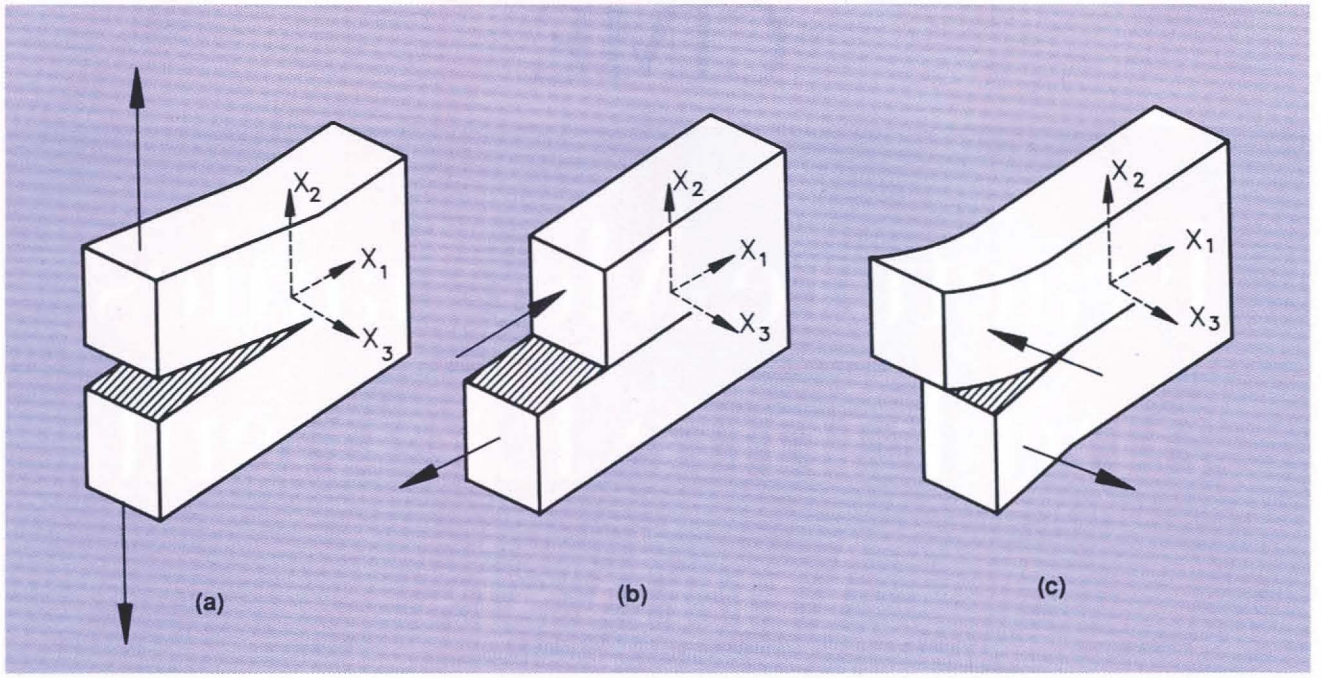


Figure 1. The three basic loading modes for fracture analysis of a cracked body: (a) mode I, opening; (b) mode II, sliding; and (c) mode III, tearing.

its center, then this change in energy per change in crack length can be expressed as

$$G = \frac{\partial U_e}{\partial a} = \frac{\pi}{E}\sigma^2 a$$

where the result of this equation is G (in honor of Griffith and defined as the crack driving energy or strain-energy release rate), E is the elastic modulus of the material, and σ is the applied stress normal to the plane of the crack. A similar expression is derived for thick plates (plane strain conditions) if $E/(1-\nu^2)$ is substituted for E in the formula (where ν is the Poisson ratio).

In the 1950s, G.R. Irwin in his analysis of the strains and stresses at the tip of a crack derived a useful expression, called the stress-intensity factor or K (for J.A. Kies, one of Irwin's collaborators). For the case of an infinite rectangular plate, $K = \sigma\sqrt{\pi a}$. If one rearranges some terms between the Griffith and Irwin analyses, then $K = \sqrt{GE}$. If G_c is then defined as the potential energy required for crack growth, then $K_c = \sqrt{G_c E}$. These equations tell us that there exists an interdependence between the applied load (σ) and the crack length (a). For example, given a certain crack length a, there will exist a certain critical applied load σ_c that will cause fracture in the structure. This approach is called the Griffith-Irwin theory of crack propagation or the linear elastic fracture mechanics approach (LEFM).

Although the mechanics of fracture are fairly straightforward, the actual fracture process on a materials basis is quite complicated. The preceding theory assumes that the material behaves in a linear elastic manner or, if plasticity does occur, it is confined to a very small region surrounding the crack tip. Most common engineering materials have some ductility, and it is expected that some plastic deformation will occur at the crack tip. However, if the material is a ceramic or if the yield strength is very near the ultimate strength (say, martensitic AISI 4340 steel), or if the specimen is thick, such that plane strain conditions are considered to exist at the crack tip, then the LEFM approach works surprisingly well.

Fracture toughness is often expressed as K_{Ic} for materials that behave in a linear elastic manner, where I refers to mode I loading conditions, i.e., the applied stress (σ) or the applied force is perpendicular to the crack plane at some remote boundary; see Figure 1(a). And since the material is assumed to behave in a linear elastic manner, the following holds:

$$K_{Ic} = \sqrt{G_{Ic}E}.$$

If significant plastic deformation does occur, then $K_{Ic} \neq G_{Ic}$ and the fracture criterion is based solely on G_{Ic}. In this latter case, the calculations for a nonlinear G are formidable; they involve accurate knowledge of the elastic/plastic behavior of the material and the numerical capability for modeling such nonlinear behavior. (For further reading on this subject, see References 2 and 3.)

Fracture typically occurs in the plane normal to the largest tensile stress and as such, K_{Ic} is commonly tabulated for many materials. For example, one compendium of fracture data is provided by Hudson and Seward [4,5]. Fracture can also occur by shearing (K_{IIc}, mode II) or by tearing (K_{IIIc}, mode III) as shown in Figures 1(b) and 1(c), respectively. However, the majority of engineering fracture mechanics work has been concerned with the effects of mode I fracture (K_{Ic}).

A subtle point about K_{Ic} measurements is that they are not directly measured. The fracture toughness test does not measure the change in surface area per change in strain energy as Griffith proposed ideally, since inelastic processes at the crack tip would be ignored. Typically, the test measures the breaking load of the fracture specimen with some prior knowledge of the initial crack size and shape. From this data, K_{Ic} is inferred based upon a standard plane strain analysis model for that particular test. The scatter in K_{Ic} measurements is often quite large; e.g., K_{Ic} values for AISI 4340 compiled from the literature ranged from 37 to 88 MPa$\sqrt{m}$.

With such large scatter among the published values for K_{Ic}, careful scrutiny should be applied to the selection of K_{Ic} data from the literature. The following must be noted: How well does it match the particular material's composition and heat treatment? Were the fracture specimens reported in the literature of similar thicknesses to those of the structure under examination? If the material tested was rolled, from what orientations relative to the principle rolling axis were the fracture specimens pre-

pared? And, if in doubt, have fracture toughness tests been performed on specimens cut from the exact material under study?

As for the fracture mechanics analysis, four requirements must be met: a crack must exist; the crack tip region should experience only a small amount of plastic yielding; K_{Ic} must be known for the modeled material; and K_I (or G_I) must be calculated and compared to K_{Ic} for failure prediction. The first three requirements are determined from inspection, mechanical property data, and fracture toughness data.

The final requirement may be simply obtained if the geometry is well defined and the loading is uncomplicated, e.g., a flat plate under tensile loading. Although numerous analytical solutions exist for two-dimensional approximations of three-dimensional geometries with various loading configurations, many times this is not the case. As such, complex engineering geometries often require that the fracture analyst numerically calculate K_I or G_I for the structure under review.

Finite Element Analysis

The routine use of fracture mechanics in design is largely feasible due to the availability of finite element analysis packages for many computer platforms. Moreover, with the advent of high-powered personal computers, many engineers now have the potential to perform "desktop" finite element analysis of complicated structures. Although numerous specialty fracture mechanics codes exist, an initial fracture mechanics assessment might require only limited modeling and analysis. Usually, obtaining such results is well within the realm of most personal computer–based finite element codes.

In a linear elastic analysis, the total potential energy (U_e) of a structure is minimized with respect to its boundary conditions to yield an equilibrium condition. In fracture mechanics, the change of this potential energy (ΔU_e) with respect to a change in crack length (Δa) is minimized. Numerically calculating G_I from a finite element analysis of any structure thereby requires U_e at one crack length, say a_1, and U_e at another crack length, say a_2. This procedure approximates G_I as $(U_{e2} - U_{e1})/(a_2 - a_1)$. Considering numerical limitations, the error in using this approach decreases as the limit of $\Delta a \rightarrow 0$.

Figure 2 shows a quarter-symmetry finite element model (FEM) of a cracked plate of uniform thickness. The mesh is graded near the crack tip with the complete model requiring 57 eight-node elements. The crack length was varied from $a_1 = 1.0$ to $a_2 = 1.1$. The strain energy can be calculated as:

$$U_e = \frac{1}{2} \sum_{i=0}^{n} k_i u_i^2$$

where k is the element stiffness and u is the displacement vector for i through n elements. If this feature is unavailable within the finite element program, then U_e can be calculated from the forces and displacements along the boundary of the structure (i.e., $U_e = F\Delta u$). Using the formula $K_I = \sqrt{G_I E}$ and comparing the result with the analytical expression for $K_I = \sigma \sqrt{\pi a}$, the error is 0.5 percent.

The usefulness of the Griffith energy method is twofold: crack tip modeling is unnecessary, and a relatively coarse finite element mesh is acceptable for calculating G_I or K_I. (Note: crack tip regions within complicated structures should be modeled using a fine mesh, typically used for a stress concentration, to fully capture the strain energy of the crack region.) Drawbacks of this technique are the necessity of remeshing the crack tip region(s) and repeating the analysis procedure.

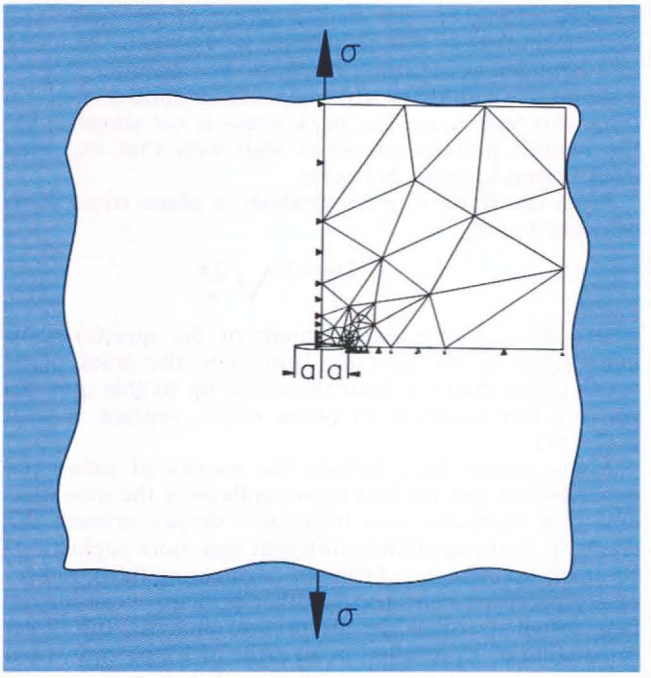


Figure 2. A center-cracked plate. By symmetry, it is necessary to model only a quarter of the plate (quarter-symmetry FEM).

One way to circumvent these problems is to calculate K_I directly from the displacements along the crack face. This method requires the finite element model to accurately reflect the near singular strains and stresses that exist at the crack tip. Theoretically, fracture is the breaking of individual atomic bonds at the tip of an advancing crack. Based on this premise, the entire loading energy of the structure is focused down to the breaking of one atomic bond. Fortunately, most materials blunt these near-infinite strains and stresses by some small amount of plastic deformation at the crack tip, resulting in a "bounded" strain/stress response.

To model this crack tip behavior, a slight modification is made to the standard 8-node quadrilateral element (the same approach can be applied to the 20-node brick element). This modification involves only the movement of the midside nodes of the elements along the crack front to their quarter-point locations (i.e, ¼L, where L is the length of that side), thus producing a stress/strain sin-

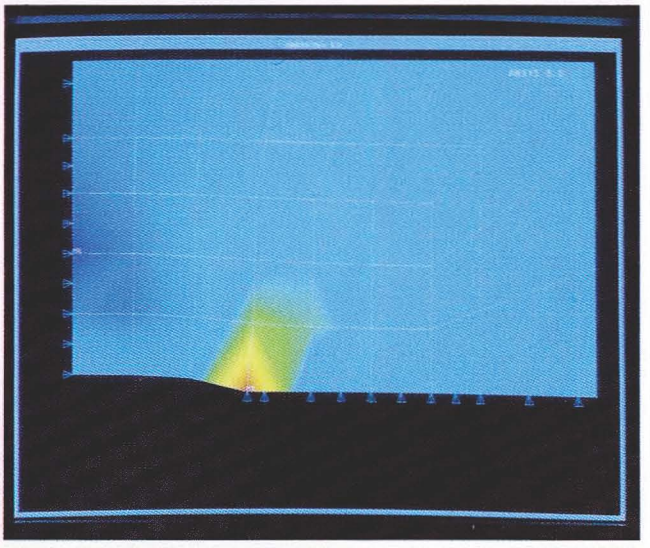


Figure 3. Von Mises stress contours with quarter-point elements at the crack tip.

gularity. However, due to the nature of the finite element formulation, this singularity is "blunted," resulting in a bounded strain/stress response. For example, Figure 3 shows the von Mises stress contours around a crack tip using this technique. The peak stress is not singular, yet the overall pattern compares well with that expected from typical material behavior.

From the finite element analysis, a plane stress K_I is calculated using

$$K_I = \frac{E(\Delta y_{qpn})}{4}\sqrt{\frac{2\pi}{r}}$$

where Δy_{qpn} is the displacement of the quarter-point node (qpn) in the direction normal to the crack plane and r is the distance from the crack tip to this qpn. To convert this equation to plane strain, replace E with $E/(1-\nu^2)$.

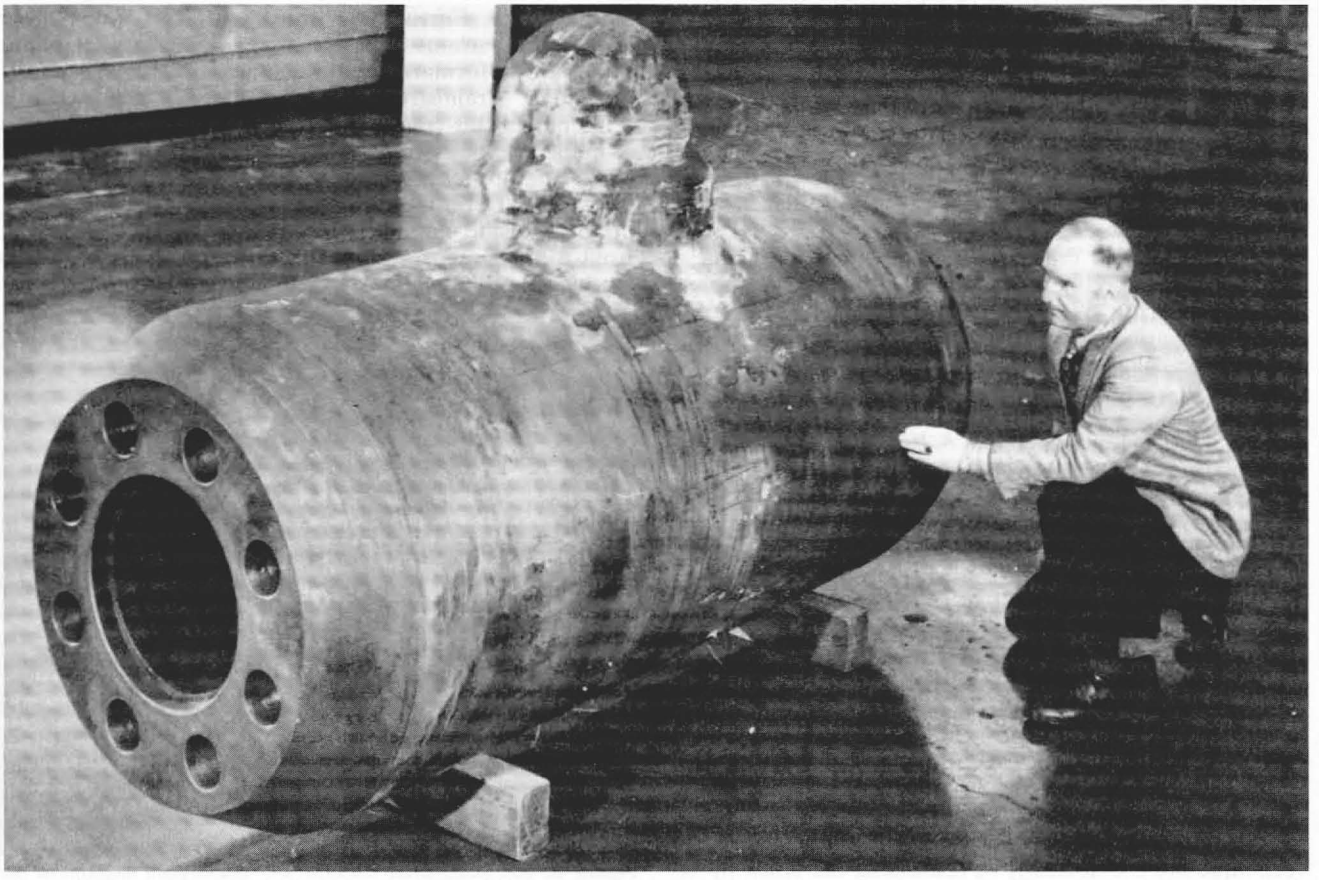
Many researchers debate the merits of using the quadrilateral qpn method versus collapsing the quadrilateral qpn elements into triangular shapes around the crack tip or using entirely different and more sophisticated crack tip elements. From an engineering basis, unnecessary precision can obscure other perhaps more important design decisions. In the present analysis, the direct calculation of K_I resulted in an error of 1.2 percent when compared to the analytical solution. When experimental values for K_{Ic} can vary an order of magnitude or more than this error, the issue of what technique to use for increased K_I precision is a decision that depends on the problem under analysis.

Both methods of evaluating K_I, the Griffith energy method and the Irwin stress-intensity method, are applicable to three-dimensional problems. If materials with different elastic moduli are used within the structure, the stress-intensity approach should be considered for both 2-D and 3-D problems. In such cases, the Griffith energy method cannot be used to find K since the mathematical relationship between G and K was derived assuming a homogeneous continuum. Lastly, the main advantage of these numerical approaches is that the engineer can apply any combination of loading conditions—displacements, forces, pressures, and thermal gradients—and still calculate G or K values with no additional effort.

Comparison Between FEM and Experiment

As a practical matter, the nuclear industry is concerned about the growth of surface flaws in nuclear containment vessels. For example, a surface flaw may propagate into the reentrant corner of a reactor pressure vessel's primary coolant pipe during severe thermal shock conditions when there is a loss of coolant within the pressure vessel. Since a full-scale thermal-shock-proof test of a reentrant corner surface crack in a boiling water reactor would be prohibitively expensive, engineers at the Oak Ridge National Laboratories (Oak Ridge, Tenn.) during the 1970s developed a scaled-down version of this pressure vessel, called the intermediate test vessel. In this scaled-down version, shown in Figure 4, the diameter and length were reduced by an order of magnitude while the wall thickness and nozzle dimensions were kept the same. The material used for the intermediate test vessel was the same as in the boiling water reactor (a nuclear-grade steel A 533B).

During the same time period, C.W. Smith and W.H. Peters [6] at Virginia Polytechnic Institute and State University (Blacksburg) and J.G. Merkle at Oak Ridge performed a series of three-dimensional stress freezing photoelastic simulation tests of the reentrant corner surface flaw.



Courtesy C.W. Smith

Figure 4. Scaled model of a boiling water reactor vessel (intermediate test vessel).

In stress freezing photoelasticity, a birefringent material is heated to a second-phase temperature, loaded, and then cooled slowly. In this manner the strains are "frozen" into the birefringent epoxy, much like tar cooled in a squeezed sponge. However, the measured photoelastic strains are elevated in magnitude by about 15 percent to those measured in steels due to a Poisson ratio effect ($\nu_{epoxy} \sim 0.5$ and $\nu_{steel} \sim 0.3$).

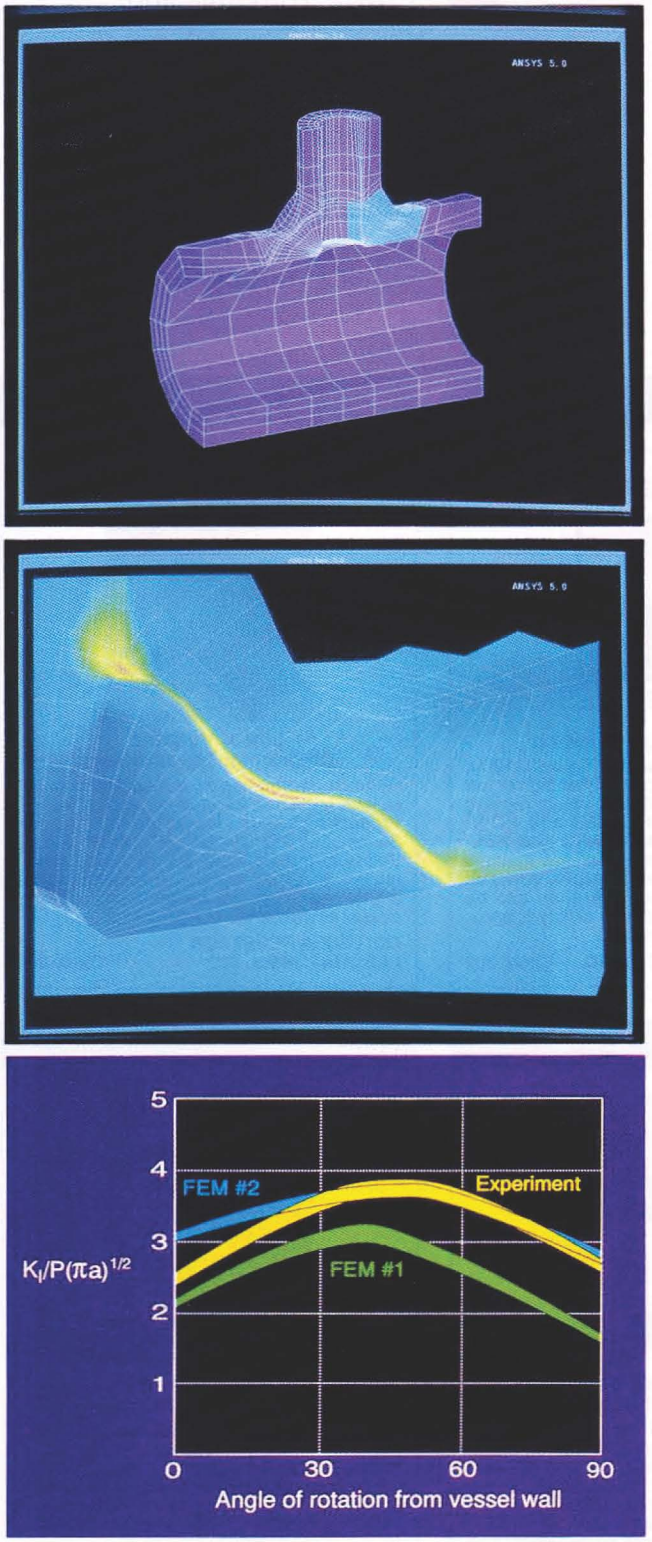


Figure 5. Half-symmetry finite element model of the intermediate test vessel *(top)* with the corresponding normalized stress-intensity results *(center)*. For comparison, the experimental photoelastic results and the more accurate FEM results from Kathiresan and Atluri are shown *(bottom)*.

To complement this experimental work, K. Kathiresan and S.N. Atluri [7] at Georgia Institute of Technology (Atlanta) conducted a detailed three-dimensional finite element analysis using a specialty hybrid displacement formation along the crack front. A simplified version of their model was constructed by the authors using qpn-modified 20-node brick elements along the crack front and 8-node bricks elsewhere. Our model is shown in Figure 5 with an expanded view of the nozzle corner crack region. Figure 5 also presents the comparison between the stress-intensity factors (K_I) generated from the photoelastic model (experiment) and the simplified (FEM 1) and Atluri's (FEM 2, Reference 8) finite element models. From Figure 5, it is clear that Atluri's approach more closely models Smith's experimental analysis; nonetheless, our simplified model captures the salient features of the photoelastic experiment considering the errors involved in measuring K_{Ic} values.

In conclusion, the engineer should realize that analytical results are only as good as the experimentally measured fracture toughness data and that the fracture toughness of a material is not single-valued but may vary significantly due to environmental or fatigue effects. Furthermore, the presented approach is quite basic and one should consider that the field of fracture mechanics is rich with analyses that did not work. This discussion has not considered the effects of fatigue, corrosion, or thermal mechanisms on the fracture process—these can play a dominant role depending on the service environment.

Nonetheless, engineering has often been a discipline of making science work and, when properly applied, the use of fracture mechanics principles and finite element analysis can be a useful tool to ensure that the engineer creates the safest, toughest, and lightest structure. ■

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